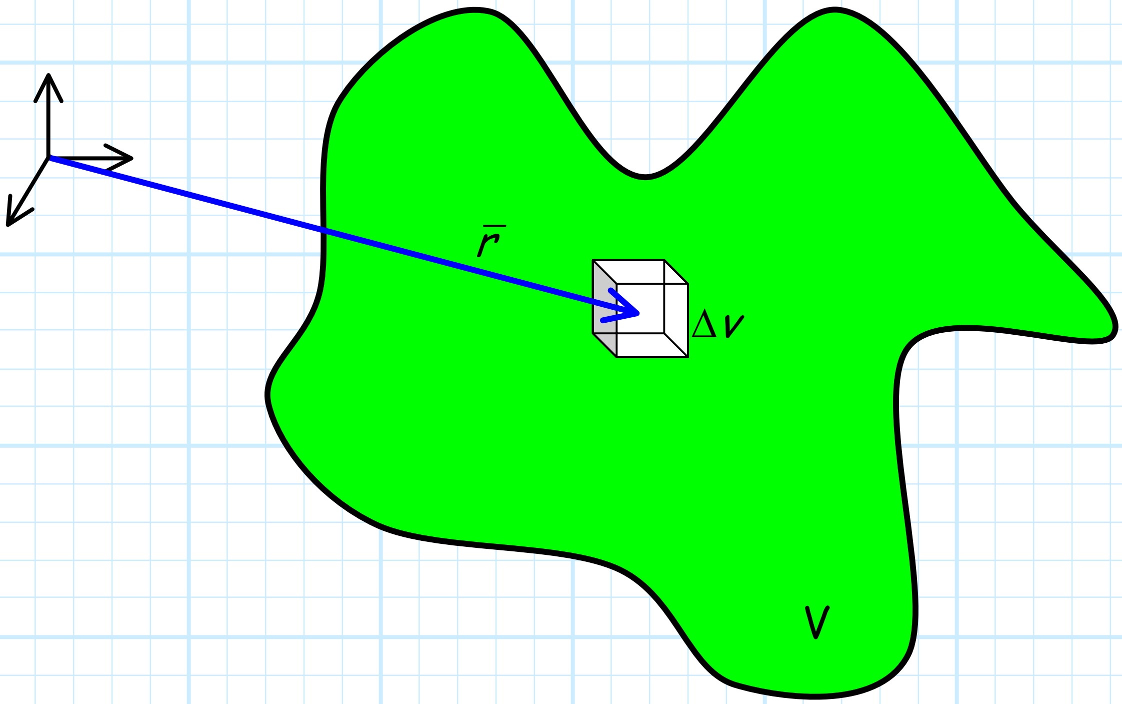
Charge Density

In many cases, charged particles (e.g., electrons, protons, positive ions) are **unevenly distributed** throughout some volume V.

We define **volume charge density** at a specific point $\bar{r}$ by evaluating the total net charge ΔQ in a small volume Δv surrounding the point.



$$\text{Volume Charge Density} = \rho_v(\bar{r}) \doteq \lim_{\Delta v \to 0} \frac{\Delta Q}{\Delta v}$$

Volume charge density is therefore a **scalar field**, and is expressed with units such as **coulombs/m^3**.

IMPORTANT NOTE: Volume charge density indicates the **net** charge density at each point $\bar{r}$ within volume V.

Q: *What exactly do you mean by **net** charge density ?*

A: Remember, there are positively charged particles and there are negatively charged particles, and **both** can exist at the same location $\bar{r}$.

Thus, a **positive** charge density does **not** mean that **no** negatively charged particles (e.g., electrons) are present, it simply means that there is **more** positive charge than there is negative!

It might be more instructive to define:

$$\Delta Q = \Delta Q^{+} + \Delta Q^{-}$$

where ΔQ^{+} is the amount of **positive** charge (therefore a **positive number**) and ΔQ^{-} is the amount of **negative** charge (therefore a **negative number**). We can call ΔQ the net, or **total charge**.

Volume charge density can therefore be expressed as:

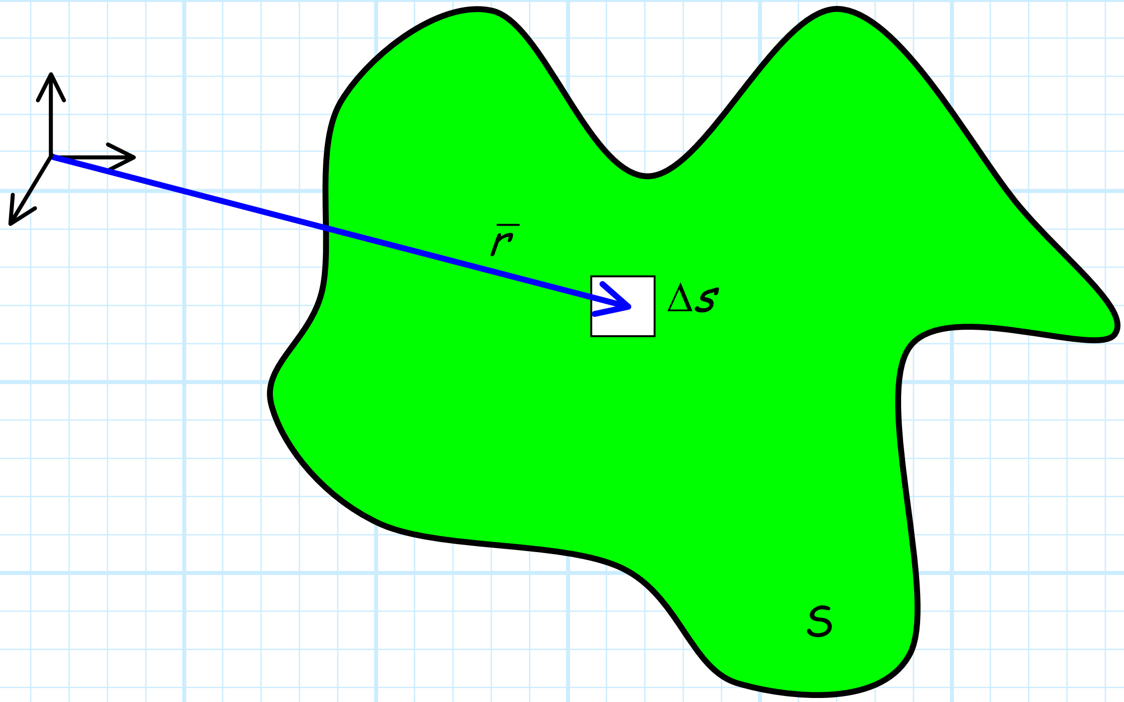
$$\rho_v(\bar{r}) \doteq \lim_{\Delta v \to 0} \frac{\Delta Q^+ + \Delta Q^-}{\Delta v} = \rho_v^+(\bar{r}) + \rho_v^-(\bar{r})$$

For example, the charge density at some location $\bar{r}$ due to negatively charged particles might be –10.0 C/m^3, while that of positively charged particles might be –5 C/m^3. Therefore, the net, or **total** charge density is:

$$\rho_v(\bar{r}) = \rho_v^+(\bar{r}) + \rho_v^-(\bar{r}) = 5 + (-10) = -5 \quad C/m^3$$

Surface Charge Density

Another possibility is that charge is unevenly distributed across some surface S. In this case, we can define a **surface charge density** as by evaluating the total charge ΔQ on a small patch of surface Δs, located at point $\bar{r}$ on surface S:



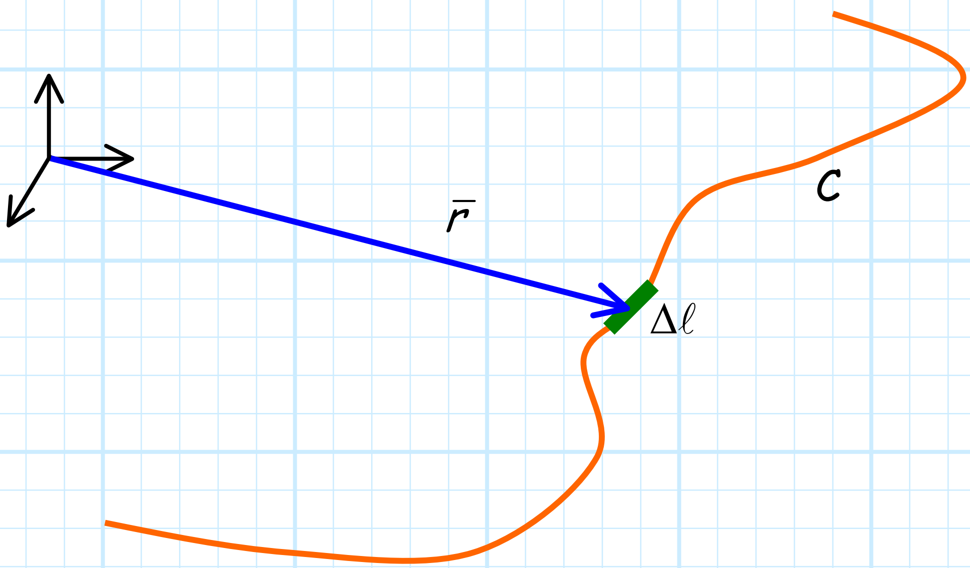
Surface charge density $\rho_s(\bar{r})$ is therefore defined as:

$$\rho_s(\bar{r}) \doteq \lim_{\Delta s \to 0} \frac{\Delta Q}{\Delta s}$$

Note the **units** for surface charge density will be **charge/area** (e.g. **C/m^2**).

Line Charge Density

Finally, we also consider the case where charge is unevenly distributed across some **contour** C. We can therefore define a **line charge density** as the charge ΔQ along a small distance $\Delta \ell$, located at point $\bar{r}$ of contour C.



We therefore define line charge density $\rho_\ell(\bar{r})$ as:

$$\rho_\ell(\bar{r}) \doteq \lim_{\Delta\ell \to 0} \frac{\Delta Q}{\Delta \ell}$$

As you might expect, the units of a line charge density is charge per length (e.g., **C/m**).